## University of Mumbai

Program: Computer Engineering
Curriculum Scheme: Rev 2019
Examination: SE Semester III
Course Code: CSC302 and Course Name: Discrete Structures and Graph Theory
Time: 2 hour

| Q1. | Choose the correct option for following questions. All the Questions are compulsory and carry equal marks |
| :---: | :---: |
| 1. | $(\mathrm{p} \rightarrow \mathrm{r}) \vee(\mathrm{q} \rightarrow \mathrm{r})$ is logically equivalent to |
| Option A: | $(\mathrm{p} \wedge \mathrm{q}) \vee \mathrm{r}$ |
| Option B: | $(\mathrm{p} \vee \mathrm{q}) \rightarrow \mathrm{r}$ |
| Option C: | $(\mathrm{p} \wedge \mathrm{q}) \rightarrow \mathrm{r}$ |
| Option D: | $(\mathrm{p} \rightarrow \mathrm{q}) \rightarrow \mathrm{r}$ |
| 2. | How many five-digit numbers can be made from the digits 1 to 7 if repetition is allowed? |
| Option A: | 16807 |
| Option B: | 54629 |
| Option C: | 23467 |
| Option D: | 32354 |
| 3. | Two sets are called disjoint if there ___ is the empty set. |
| Option A: | Union |
| Option B: | Intersection |
| Option C: | Complement |
| Option D: | Difference |
| 4. | A sub lattice (say, S) of a lattice(say, L) is a convex sub lattice of L if |
| Option A: | $x>=z$, where $x$ in $S$ implies $z$ in $S$, for every element $x, y$ in $L$ |
| Option B: | $x=y$ and $\mathrm{y}<=\mathrm{z}$, where $\mathrm{x}, \mathrm{y}$ in S implies z in S, for every element $\mathrm{x}, \mathrm{y}, \mathrm{z}$ in L |
| Option C: | $\mathrm{x}<=\mathrm{y}<=\mathrm{z}$, where $\mathrm{x}, \mathrm{y}$ in S implies z in S, for every element $\mathrm{x}, \mathrm{y}, \mathrm{z}$ in L |
| Option D: | $x=y$ and $y>=z$, where $x, y$ in S implies $z$ in $S$, for every element $x, y, z$ in $L$ |
| 5. | The inclusion of $\qquad$ sets into $R=\{\{1,2\},\{1,2,3\},\{1,3,5\},\{1,2,4\},\{1,2$, $3,4,5\}\}$ is necessary and sufficient to make R a complete lattice under the partial order defined by set containment. |
| Option A: | \{1\}, $\{2,4\}$ |
| Option B: | \{1\} |
| Option C: | \{1\}, \{1, 2, 3\} |
| Option D: | $\{1\},\{1,3\},\{1,2,3,4\},\{1,2,3,5\}$ |
| 6. | If $A$ and $B$ are sets and $A \cup B=A \cap B$, then |
| Option A: | $\mathrm{A}=\Phi$ |
| Option B: | $\mathrm{B}=\Phi$ |
| Option C: | $\mathrm{A}=\mathrm{B}$ |
| Option D: | $\mathrm{A} \subseteq \mathrm{B}$ |


| 7. | The compound propositions p and q are called logically equivalent if $\qquad$ is a tautology. |
| :---: | :---: |
| Option A: | $\mathrm{p} \leftrightarrow \mathrm{q}$ |
| Option B: | $\mathrm{p} \rightarrow \mathrm{q}$ |
| Option C: | $\neg(p \vee q)$ |
| Option D: | $\neg p \vee \neg q$ |
|  |  |
| 8. | If every element of a group G is its own inverse, then G is |
| Option A: | finite |
| Option B: | infinite |
| Option C: | cyclic |
| Option D: | Abelian |
|  |  |
| 9. | Consider the binary relation, $\mathrm{A}=\{(\mathrm{a}, \mathrm{b}) \mid \mathrm{b}=\mathrm{a}-1$ and $\mathrm{a}, \mathrm{b}$ belong to $\{1,2,3\}\}$. The reflexive transitive closure of A is? |
| Option A: | $\{(\mathrm{a}, \mathrm{b}) \mid \mathrm{a}>=\mathrm{b}$ and $\mathrm{a}, \mathrm{b}$ belong to $\{1,2,3\}\}$ |
| Option B: | $\{(\mathrm{a}, \mathrm{b}) \mid \mathrm{a}>\mathrm{b}$ and $\mathrm{a}, \mathrm{b}$ belong to $\{1,2,3\}\}$ |
| Option C: | $\{(\mathrm{a}, \mathrm{b}) \mid \mathrm{a}<=\mathrm{b}$ and $\mathrm{a}, \mathrm{b}$ belong to $\{1,2,3\}\}$ |
| Option D: | $\{(\mathrm{a}, \mathrm{b}) \mid \mathrm{a}=\mathrm{b}$ and $\mathrm{a}, \mathrm{b}$ belong to $\{1,2,3\}\}$ |
|  |  |
| 10. | Let $f$ and $g$ be the function from the set of integers to itself, defined by $f(x)=2 x+$ 1 and $g(x)=3 x+4$. Then the composition of $f$ and $g$ is $\qquad$ |
| Option A: | $6 x+7$ |
| Option B: | $6 x+6$ |
| Option C: | $6 x+8$ |
| Option D: | $6 x+9$ |
|  |  |
| 11. | An algebraic structure ___ is called a semigroup. |
| Option A: | (P, *) |
| Option B: | (Q, +, *) |
| Option C: | (P, +) |
| Option D: | (+, *) |
|  |  |
| 12. | Solve using warshall's algorithm $\mathrm{R}=\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{c})\}$ defined of A where $\mathrm{A}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ |
| Option A: | \{(a,a),(c,c),(b,a),(b,b),(b,c)\} |
| Option B: | \{(a,a),(a,b),(a,c), (b,c)\} |
| Option C: | $\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{a}, \mathrm{c}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b}),(\mathrm{b}, \mathrm{c}),(\mathrm{c}, \mathrm{a}),(\mathrm{c}, \mathrm{b})\}$ |
| Option D: | $\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{a}, \mathrm{c}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b}),(\mathrm{b}, \mathrm{c})$ \} |
|  |  |
| 13. | The number of symmetric relations on a set with 15 distinct elements is |
| Option A: | 2196 |
| Option B: | 250 |
| Option C: | 2320 |
| Option D: | 278 |
|  |  |
| 14. | A cyclic group is always |
| Option A: | abelian group |
| Option B: | monoid |
| Option C: | semigroup |


| Option D: | subgroup |
| :---: | :---: |
| 15. | If the longest chain in a partial order is of length 1 , then the partial order can be written as $\qquad$ disjoint antichains. |
| Option A: | 12 |
| Option B: | 1+1 |
| Option C: | 1 |
| Option D: | 11 |
|  |  |
| 16. | Warshall's Algorithm is used to find ___closure |
| Option A: | Transitive |
| Option B: | Symmetric |
| Option C: | Asymmetric |
| Option D: | Reflexive |
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| 17. | and ___ are the two binary operations defined for lattices. |
| Option A: | Join, meet |
| Option B: | Addition, subtraction |
| Option C: | Union, intersection |
| Option D: | Multiplication, modulo division |
|  |  |
| 18. | In a group of 300 persons, 160 drink tea and 170 drink coffee, 80 of them drink both, How many persons do not drink either? |
| Option A: | 40 |
| Option B: | 45 |
| Option C: | 50 |
| Option D: | 60 |
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| 19. | How many properties can be held by a group? |
| Option A: | 2 |
| Option B: | 3 |
| Option C: | 5 |
| Option D: | 4 |
|  |  |
| 20. | Suppose S is a finite set with 7 elements. How many elements are there in the largest equivalence relation on S ? |
| Option A: | 100 |
| Option B: | 56 |
| Option C: | 49 |
| Option D: | 78 |


| Q2 <br> (20 Marks Each) |  |
| :---: | :--- |
| A | Solve any Two Questions out of Three |
| i. | How many four digits can be formed out of digits 1,2,3,5,7,8,9 marks each no digits <br> repeated twice? How many of these will be greater than 3000? |
| ii. | Let $A=\{1,2,3,4,5\}$ and let <br> $R=\{(1,1),(1,3),(1,4),(2,2),(2,5),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,2),(5,5)\}$. |


|  | Check if $R$ is a equivalence relation. Justify your answer. Find equivalence <br> classes of $A$. |
| :---: | :--- |
| iii. | What is the solution of the recurrence relation an= $-a n-1+4 a n-2+4 a n-3$ <br> with $a 0=8, a 1=6$ and $a 2=26 ?$ |


| Q3. <br> (20 Marks Each) | Solve any Two Questions out of Three <br> A <br> i. <br> ii. <br> Find the number of positive integers not exceeding 100 that are not <br> divisible by 5 or 7. Also draw corresponding Venn diagram.A travel company surveyed its travelers, to learn how much of their travel <br> is taken with an Airplane, a Train or a car. The following data is known; <br> make a complete Venn Diagram with all the data. The number of people <br> who flew was 1307. The number of people who both flew and used a train <br> was 602. The people who used all three were 398 in number. Those who <br> flew but didn't drive came to total 599. Those who drove but did not use <br> train totaled 1097. There were 610 people who used both trains and cars. <br> The number of people who used either a car or train or both was 2050. <br> Lastly, 421 people used none of these .Find out how many people drove <br> but used neither a train nor an airplane, and also, how many people were <br> in the entire survey. |
| :---: | :--- |
| iii. | Prove that set G=\{1,2,3,4,5,6\} is a finite abelian group of order 6 w.r.t <br> multiplication module 7. |

